TRIGONOMETRY GUIDE

By:

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(Bridge Course for Students Moving from Class 10 to 11)

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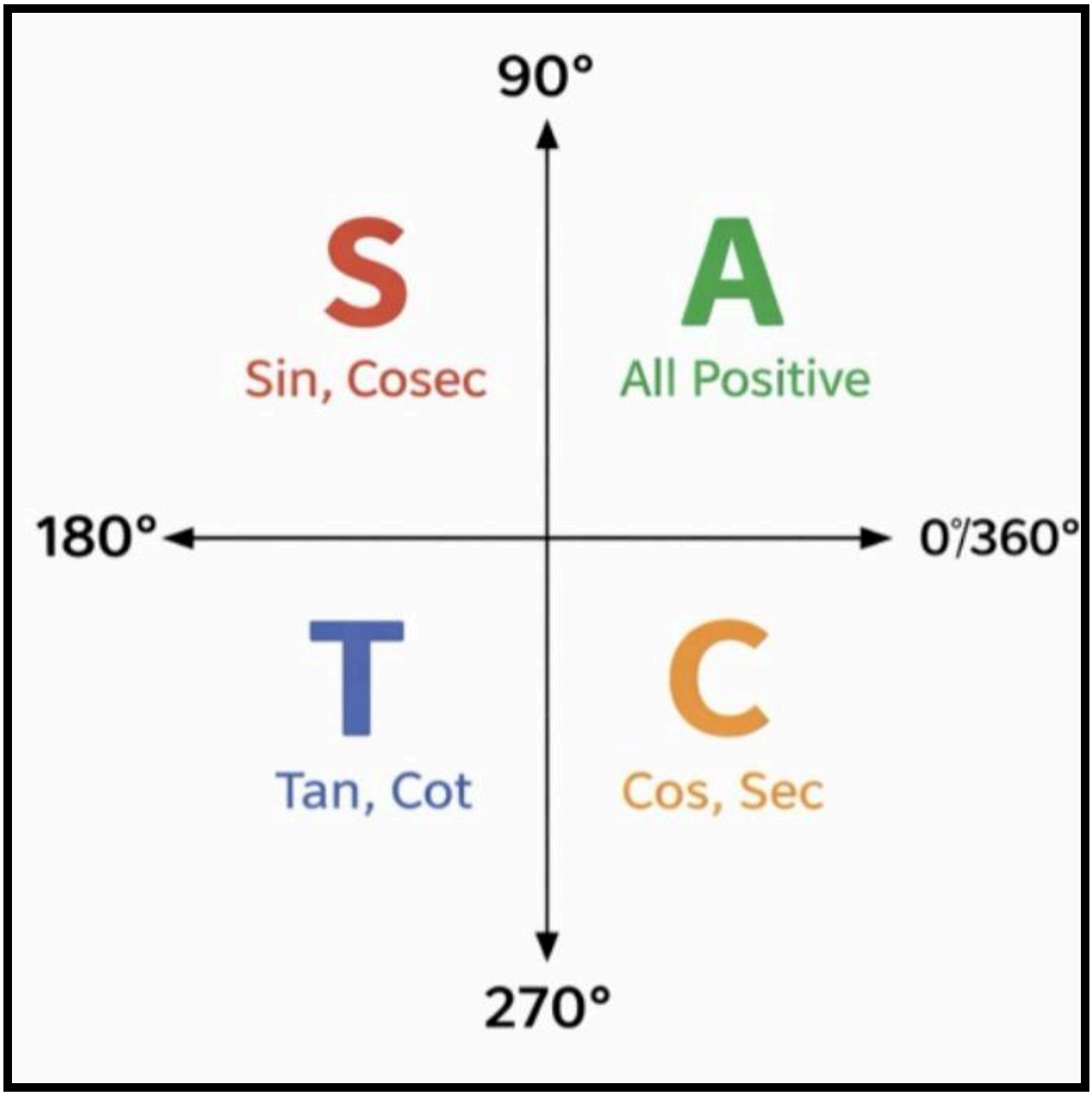

90°
S
Sin, Cosec
A
All Positive
180°
0°/360°
T
Tan, Cot
C
Cos, Sec
270°

5 Brief points on ASTC rule

- This rule helps decide the sign (+/-) of [Sin, cos, tan, cot, cosec, sec] for any angle
- 'A' (1st quadrant) :- All Trigonometric Ratios are positive
- 'S' (2nd quadrant) :- only [Sin, cosec] are positive
- 'T' (3rd quadrant) :- only [tan, cot] are positive
- 'C' (4th quadrant) :- only [cos, sec] are positive

Odd functions :

1) $\sin(-x) = -\sin x$

2) $\tan(-x) = -\tan x$

3) $\operatorname{cosec}(-x) = -\operatorname{cosec} x$

4) $\cot(-x) = -\cot x$

Even functions :

- "cos" is an even function $\rightarrow [\cos(-x) = \cos x]$
  So Sign doesn't change

- "Sec" is Reciprocal of cos $\rightarrow \left[\sec(-x) = \frac{1}{\cos(-x)}\right]$

  $\frac{1}{\cos(-x)} = \frac{1}{\cos x} = \sec x$, $\therefore \sec(-x) = \sec x$

- Since (cos, sec) doesn't change for $(-x)$

Compound
Angles

1) Compound angles means an angle formed by adding or subtracting two angles
$(x+y, x-y)$

2) It is used when the given angle is not standard (not $30°, 45°, 60°$, etc)

3) We need split the angle into two known angles
example :- $(75° = 45° + 30°)$, $(15° = 45° - 30°)$

4) each part (x and y) must be an angle, which (Sin, cos, tan, cot) values you already know

5) After Splitting, you combine the Trigonometry values of both angles using compound-angle Identities

6) Main Idea :-
- Break angle (Split angle into two)
- apply compound-angle Identity
- Substitute known values
- Simplify

example Splits :-
- $(15° = 45° - 30°)$
- $(75° = 45° + 30°)$
- $(105° = 60° + 45°)$
- $(165° = 180° - 15°)$

FORMULA'S :-

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

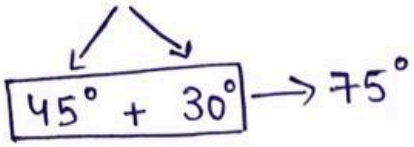
$$\cot(x+y) = \frac{\cot y \cot x - 1}{\cot y + \cot x}$$

$$\cot(x-y) = \frac{\cot y \cot x + 1}{\cot y - \cot x}$$

EXAMPLE :-

1) $\sin 75^\circ$

A) Split 75° into Two Known angles



$\rightarrow \sin 75^\circ = \sin(45^\circ + 30^\circ)$

$\sin(x+y) = \sin A \cos B + \cos A \sin B$

$\rightarrow \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

(keep the values) $\leftarrow = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

(Simplify) $\leftarrow$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ $\rightarrow$ Answer

2) $\cos 15^\circ$

A) $\cos 15^\circ$ $(45^\circ - 30^\circ)$

$\Rightarrow \cos(45^\circ - 30^\circ)$

$\Rightarrow$ **$\cos(x-y) = \cos x \cos y + \sin x \sin y$**

$\Rightarrow \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ → Answer

So, we can conclude that

$\boxed{\sin 75^\circ = \cos 15^\circ}$

3) $\sin(m+1)x \sin(m+2)x + \cos(m+1)x \cos(m+2)x = \cos x$ (prove LHS = RHS)

A) LHS:-

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

LHS expression is the formula of $\cos(x-y)$

$\sin(m+1)x \sin(m+2)x + \cos(m+1)x \cos(m+2)x$

$\Rightarrow \cos((m+1)x - (m+2)x)$

$\Rightarrow \cos(mx + x - mx - 2x)$

$\Rightarrow \cos(mx + x - mx - 2x)$

$\Rightarrow \cos(x - 2x)$

$\Rightarrow \cos(-x)$

$\Rightarrow \cos x$ LHS = RHS

Exercise – 1 (Easy)

1) Find the exact values of:

a) $\sin 135°$
b) $\cos 150°$
c) $\tan 105°$
d) $\cot 15°$
e) $\cot 120°$
f) $\cos 75°$

2) Prove:

a) $\sin 75° = \cos 15°$
b) $\sin 120° = \cos 30°$
c) $\tan 105° = -\cot 15°$
d) $\sin 45° \cos 45° + \cos 45° \sin 45° = 1$

3) Simplify:

a) $\cos 60° \cos 30° + \sin 60° \sin 30°$

b) $(\sin 85° \cos 5° - \cos 85° \sin 5°) / \sin 80°$

c) $(\sin 40° \cos 50° + \cos 40° \sin 50°) / \cos 0°$

Exercise – 2 (Medium)

1) Prove:

a) $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$
b) $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$
c) $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$
d) $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$

2) Simplify:

a) $\sin(A + B) \sin(A - B)$

b) $\cos(A + B) \cos(A - B)$

c) $(\cos 95° \cos 5° - \sin 95° \sin 5°) / \sin 100°$

3) Evaluate:

a) If $A = 30°$, $B = 45°$, evaluate
$\sin A \cos B + \cos A \sin B$

b) If $A = 60°$, $B = 30°$, evaluate
$\cos A \cos B + \sin A \sin B$

Exercise – 3 (Hard)

1) Simplify:

a) $\sin^2 75° - \sin^2 15°$

b) $\sin^2 37½° - \sin^2 7½°$

c) $\sin^2 82½° - \sin^2 22½°$

Use formula:
$\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$

2) Prove:

a) $\sin (A + B) / \cos A = \tan A \cos B + \sin B$

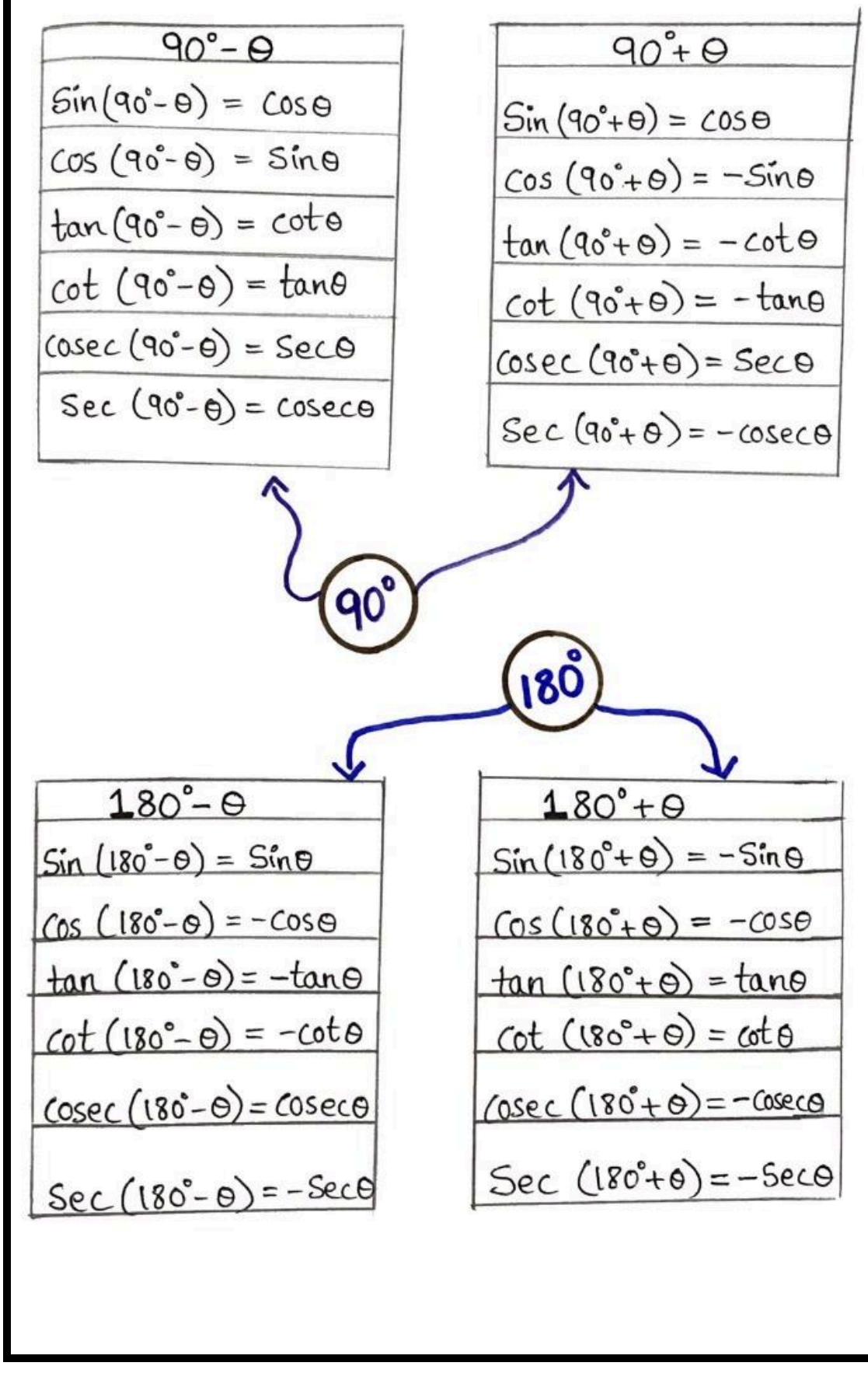
b) $(\sin (A + B) + \sin (A - B)) / (\cos (A + B) + \cos (A - B)) = \tan A$

c) $\sin (x + y) / \cos (x + y) = (\tan x + \tan y) / (1 - \tan x \tan y)$

Allied
Angles

Use of ALLIED ANGLES

- Convert angles Outside $0°-90°$ to acute Reference angles
- Determine Sign of Trigonometric functions Using quadrant Rules (ASTC)
- Evaluate values like $\cos 210°$, $\sin 330°$, $\tan 420°$, $\cot 300°$ etc



$90^\circ - \theta$
$\sin(90^\circ - \theta) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$
$\tan(90^\circ - \theta) = \cot\theta$
$\cot(90^\circ - \theta) = \tan\theta$
$\operatorname{cosec}(90^\circ - \theta) = \sec\theta$
$\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

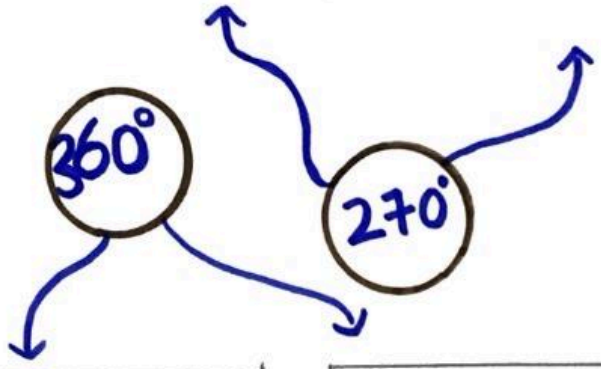
$90^\circ + \theta$
$\sin(90^\circ + \theta) = \cos\theta$
$\cos(90^\circ + \theta) = -\sin\theta$
$\tan(90^\circ + \theta) = -\cot\theta$
$\cot(90^\circ + \theta) = -\tan\theta$
$\operatorname{cosec}(90^\circ + \theta) = \sec\theta$
$\sec(90^\circ + \theta) = -\operatorname{cosec}\theta$

$180^\circ - \theta$
$\sin(180^\circ - \theta) = \sin\theta$
$\cos(180^\circ - \theta) = -\cos\theta$
$\tan(180^\circ - \theta) = -\tan\theta$
$\cot(180^\circ - \theta) = -\cot\theta$
$\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec}\theta$
$\sec(180^\circ - \theta) = -\sec\theta$

$180^\circ + \theta$
$\sin(180^\circ + \theta) = -\sin\theta$
$\cos(180^\circ + \theta) = -\cos\theta$
$\tan(180^\circ + \theta) = \tan\theta$
$\cot(180^\circ + \theta) = \cot\theta$
$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$
$\sec(180^\circ + \theta) = -\sec\theta$

$270^\circ - \theta$
$\sin(270^\circ - \theta) = -\cos\theta$
$\cos(270^\circ - \theta) = -\sin\theta$
$\tan(270^\circ - \theta) = \cot\theta$
$\cot(270^\circ - \theta) = \tan\theta$
$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$
$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$

$270^\circ + \theta$
$\sin(270^\circ + \theta) = -\cos\theta$
$\cos(270^\circ + \theta) = \sin\theta$
$\tan(270^\circ + \theta) = -\cot\theta$
$\cot(270^\circ + \theta) = -\tan\theta$
$\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$
$\sec(270^\circ + \theta) = \operatorname{cosec}\theta$



$360^\circ - \theta$
$\sin(360^\circ - \theta) = -\sin\theta$
$\cos(360^\circ - \theta) = \cos\theta$
$\tan(360^\circ - \theta) = -\tan\theta$
$\cot(360^\circ - \theta) = -\cot\theta$
$\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta$
$\sec(360^\circ - \theta) = \sec\theta$

$360^\circ + \theta$
$\sin(360^\circ + \theta) = \sin\theta$
$\cos(360^\circ + \theta) = \cos\theta$
$\tan(360^\circ + \theta) = \tan\theta$
$\cot(360^\circ + \theta) = \cot\theta$
$\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec}\theta$
$\sec(360^\circ + \theta) = \sec\theta$

SUMS :-

1) $\sin 300^\circ$

A) $\sin 300^\circ$

$\downarrow$

Check 300° is closer to $(90^\circ, 180^\circ, 270^\circ, 360^\circ)$

Since it is closer to 360°

$\rightarrow 300^\circ = (360^\circ - 60^\circ) = \sin(360^\circ - 60^\circ)$

formula $\rightarrow$ $\boxed{\sin(360^\circ - \theta) = -\sin\theta}$

$\Rightarrow \sin(360^\circ - 60^\circ) = -\sin 60^\circ$

$\Rightarrow \left(-\frac{\sqrt{3}}{2}\right) \longrightarrow$ answer

2) $\cos 330^\circ$

A) $\cos 330^\circ$

↓

closer to (360°)

$\cos(330^\circ) = \cos(360^\circ - \theta) = \cos(360^\circ - 30^\circ)$

formula $= \cos(360^\circ - \theta) = \cos\theta$

$= \cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$ → answer

3) The value of :-

$$\cos(270^\circ+x)\cos(360^\circ+x)\left[\cot(270^\circ-x)+\cot(360^\circ+x)\right]$$

A) On Applying all the formulas, The expression will be :-

$\Rightarrow \sin x \cos x [\tan x + \cot x]$

$\Rightarrow$ change the expression into sin, cos

$\Rightarrow \sin x \cos x \left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right]$

$\Rightarrow \sin x \cos x \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right]$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cancel{\sin x \cos x}\left[\frac{1}{\cancel{\sin x \cos x}}\right]$

$\Rightarrow$ (1) $\rightarrow$ answer

Exercise – 1 (Easy)

1) Find the values of:

a) $\sin 315°$
b) $\cos 210°$
c) $\tan 270°$
d) $\cot 330°$
e) $\sec 450°$
f) $\operatorname{cosec} 240°$

2) Simplify:

a) $\sin(180° - \theta) + \cos(180° - \theta)$

b) $\sin(90° - \theta) - \cos(90° - \theta)$

c) $\sin(90° - x) / \cos(90° - x)$

d) $\sin(90° - x) \sin x + \cos(90° - x) \cos x$

Exercise 2 (Medium)

Simplify:

a) $\sin(180° - 30°) + \cos(180° - 60°)$

b) $\sin(90° - \theta)\sin(90° + \theta) + \cos(90° - \theta)\cos(270° + \theta)$

c) $(\sec(270° + \theta)\sin(180° - \theta))^2$

d) $\cos(360° - \theta)\sin(360° + \theta)(\sin\theta\cos\theta)^{-1}$

e) $(\sec(360° + \theta)\cos(360° + \theta))^2$

Prove:

1. $(\tan(360° + \theta)\cot(360° + \theta))(\sec(360° + \theta)\cos(360° + \theta)) = 1$
2. $\cos(270° + \theta)\sin(360° + \theta) \times \sin^{-2}\theta = 1$
3. $\sec(360° - \theta)\sec(360° + \theta)\cos(360° + \theta)\cos(360° - \theta) = 1$

Exercise-3 (Hard)

Prove:

a) $(\cos(180° + x)\cos(-x)) / (\sin(180° - x)\cos(90° + x)) = \cot^2 x$

b) $[(\tan(270° - x)\cot(270° - x)) / (\tan(180° + x)\cot(180° + x))]^2 = 1$

c) $[\cos(270° + \theta)\sec(270° + \theta)] / [\sin(360° + \theta)\operatorname{cosec}(360° + \theta)] = 1$

Simplify:

a) $\cos1° \cdot \cos2° \cdot \cos3° \cdot \ldots \cdot \cos2410°$

b) If $A + B = 90°$, find the value of
$\tan A \cdot \tan B$

c) $\tan1° \cdot \tan2° \cdot \tan3° \cdot \ldots \cdot \tan89°$

Radians and Degrees: Brief Points

• Angles can be measured in two units: degrees and radians.

• The relation between them is:
π radians = 180°

• From this relation:
$\pi/2 = 90°$
$\pi/3 = 60°$
$\pi/4 = 45°$
$\pi/6 = 30°$

• One complete revolution:
2π radians = 360°

• Trigonometric values remain the same in both units:
$\sin(\pi/2) = \sin 90° = 1$
$\cos(\pi) = \cos 180° = -1$
$\tan(\pi/4) = \tan 45° = 1$

• Radians are mainly used in Class 11 and higher mathematics.

Thank you for choosing this Trigonometry Guide. This book was created to help students build a strong foundation while moving from Class 10 to Class 11. I hope the concepts, examples, and practice questions make learning easier and more effective.

Thank you.

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